Multi-objective Optimisation by Co-operative Co-evolution

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Abstract. This paper presents the integration between a co-operative co-evolutionary genetic algorithm (CCGA) and four evolutionary multi-objective optimisation algorithms (EMOAs): a multi-objective genetic algorithm (MOGA), a niched Pareto genetic algorithm (NPGA), a non-dominated sorting genetic algorithm (NSGA) and a controlled elitist non-dominated sorting genetic algorithm (CNSGA). The resulting algorithms can be referred to as co-operative co-evolutionary multi-objective optimisation algorithms or CCMOAs. The CCMOAs are benchmarked against the EMOAs in seven test problems. The first six problems cover different characteristics of multi-objective optimisation problems, namely convex Pareto front, non-convex Pareto front, discrete Pareto front, multimodality, deceptive Pareto front and non-uniformity of solution distribution. In contrast, the last problem is a two-objective real-world problem, which is generally referred to as the continuum topology design. The results indicate that the CCMOAs are superior to the EMOAs in terms of the solution set coverage, the average distance from the non-dominated solutions to the true Pareto front, the distribution of the non-dominated solutions and the extent of the front described by the non-dominated solutions.

1 Introduction

Genetic algorithms (GAs) have a unique niche in the area of multi-objective optimisation. Due to the parallel search nature of the algorithms, the approximation of multiple Pareto optimal solutions can be effectively executed. Various genetic algorithms are currently available for use in multi-objective optimisation [1–8]. Similar to the case of single-objective optimisation, as the search space or problem size increases, the performance of the algorithms always degrade. As a result, the non-dominated solutions identified by the algorithms may deviate from the true Pareto front. In addition, the coverage of the Pareto front by the solutions generated may also be affected. A number of strategies have been used to solve the problem. Examples of the strategies that have been successfully embedded into the algorithms include elitism [6–8], diversity control [9] and co-evolution [10, 11]. Although the strategies mentioned can be used with almost all genetic algorithms designed for multi-objective optimisation, these strategies have rarely been used with more than one algorithm and hence the effect of the same strategy on different genetic algorithms cannot be determined directly. In particular, the strategy interested for a detailed study in this paper is the co-operative co-evolutionary strategy.

A co-evolutionary search involves the use of multiple species as the representation of a solution to the optimisation problem. Each species can either compete or co-operate during the search evolution. One particular algorithm that stands out as one of the algorithms that truly exploit the concept of co-evolution is a co-operative co-evolutionary genetic algorithm or CCGA [12, 13]. In brief, a species member in the CCGA represents a part of the decision variable set where all species will co-operatively produce a complete solution to the problem. Each species member will then independently evolve using a standard genetic algorithm. By partitioning the problem in this manner, the search space that each sub-population has to cover would significantly reduce. Although the CCGA is originally developed for use in single-objective optimisation, the co-operative co-evolutionary effect has also been successfully embedded into a multi-objective genetic algorithm [10]. Keerativuttitumrong et al. [10] have proven that the multi-objective co-operative co-evolutionary genetic algorithm or MOCCGA outperforms the original multi-objective genetic algorithm or MOGA [2, 5] in six test problems introduced by Zitzler et al. [14]. As suggested by Keerativuttitumrong et al. [10], the effects of co-operative co-evolution in other genetic algorithms that are designed for use in multi-objective optimisation will be investigated in this paper. The candidate algorithms for the study include a niched Pareto genetic algorithm or NPGA [3], a non-dominated sorting genetic algorithm or NSGA [4] and a controlled elitist non-dominated sorting genetic algorithm or CNSGA [7]. In addition, the modification on the MOCCGA for a further performance enhancement will also be covered in this paper. All modified algorithms will be benchmarked against the original algorithms by means of a performance comparison. The benchmark problems used include six test problems proposed by Zitzler et al. [14] and an engineering problem called a topology design [15].

The organisation of this paper is as follows. In section 2, the genetic algorithm integration and additional genetic operators used for the performance enhancement will be discussed. The test problems and the performance evaluation criteria will be described in sections 3 and 4, respectively. In section 5, the benchmarking results and discussions are given. Finally, the conclusions are drawn in section 6.

2 GA Integration and Additional Genetic Operators

The CCGA will be integrated with each one of the four genetic algorithms for multi-objective optimisation. In order to enhance the combined algorithms, two additional genetic operators are utilised: a crowding distance selection [7, 8] and an elitist strategy. The algorithm integration and the elitist strategy are described as follows.

2.1 Genetic Algorithm Integration

Similar to the original CCGA, the population of its multi-objective counterpart also contains a number of species or sub-populations where each species represents a decision variable or a part of solution. The objective vector of a species member or an individual is obtained after combining it with the remaining species extracted from a non-dominated solution, which is randomly picked from a preserved non-dominated solution set. It is noted that the preserved non-dominated solution set is initially created by combining different species together randomly and choosing only the non-dominated solutions. If the complete solution obtained after combining the individual of interest with other species is neither dominated by any solutions in the preserved set nor a duplicate of a solution in the preserved set, then this complete solution will be added to the preserved set. At the same time, if the newly created solution dominates any existing solutions in the preserved set, the dominated solutions will be expunged from the set. In order to maintain the diversity within the preserved non-dominated solution set, the crowding distance selection [7, 8] is used to regulate the size of the preserved set. After the objective values have been assigned to every individual in all sub-populations, the evolution of every species is then commenced by one of the four genetic algorithms for multi-objective optimisation. The resulting co-operative co-evolutionary multi-objective optimisation algorithms (CCMOAs) can be uniquely referred to as a co-operative co-evolutionary multi-objective genetic algorithm (CCMOGA), a co-operative co-evolutionary niched Pareto genetic algorithm (CCNPGA), a co-operative co-evolutionary non-dominated sorting genetic algorithm (CCNSGA) and a co-operative co-evolutionary controlled elitist non-dominated sorting genetic algorithm (CCCNSGA). The name given to the resulting algorithm obtained after combining the MOGA with the CCGA in this paper differs from that used by Keerativuttitumrong et al. [10] since the integration protocol is quite different.

2.2 Elitist Strategy

Elitism has been proven to be an important part in the success of multi-objective optimisation using a genetic algorithm [7, 8]. Since the use of an elitist strategy has not been mentioned in the original publications of MOGA, NPGA and NSGA, such a strategy will be used in conjunction with the MOGA, CCMOGA, NPGA, CCNPGA, NSGA and CCNSGA. Similar to the case of single-objective genetic algorithm, the implemented elitist strategy involves passing a number of individuals (of the same species) from one generation to the next without either crossover or mutation. However, the elite individuals in this case will be the non-duplicated non-dominated individuals. The prevention of using duplicated individuals as elite individuals would promote genetic diversity [16, 17]. Note that if the number of non-duplicated non-dominated individuals acquired exceeds a preset limit, the crowding distance selection will be used to select the individuals for the elite individual set.

3 Test Problems

In order to assess the performance of the four combined algorithms, they will be benchmarked using six optimisation test cases developed by Zitzler et al. [14] and an engineering problem called a topology design [15]. The optimisation problems T_1–T_6 proposed by Zitzler et al. [14] are minimisation problems with m decision variables and two objectives. T_1 is a 30-dimensional problem with a convex Pareto front, which is continuous and uniformly distributed. T_2 is also a 30-dimensional problem but has a non-convex Pareto front. T_3 is a 30-dimensional problem with five discrete Pareto fronts. T_4 is a 10-dimensional problem with 21^9 local Pareto fronts and therefore is used to test the algorithm's ability to deal with multi-modality. T_5 is an 11-dimensional problem with deceptive Pareto fronts. T_6 is a 10-dimensional problem with the non-uniform search space.

In contrast, a 2D heat transfer problem is used as a real life case study for topology design optimisation [15]. Given a wall with linear distributed temperature profile, a limited space is available for attaching a solid protruding configuration that facilitates heat loss from the wall as shown in Fig. 1 (left). The optimisation aims to obtain lightweight configuration with high dissipated heat from the wall and body into the surroundings. The boundary between the protruding body and the environment is assumed to be convective with very good air circulation that the ambient air temperature in close proximity to the wall and the protruding body remains unchanged. Domains of the available space are dividing into uniform grids with 10 rows and 10 columns. The problem is thus encoded using a 100-bit binary representation in the on-off style. The block insertion on a grid is represented by '1' whilst '0' signifies a void in the corresponding location. A complete solution can be divided into 5 parts, each containing 20 bits that represents 2 rows in the grid. The heat dissipation performance of each configuration is numerically evaluated by the finite volume simulation and, thus, the true Pareto front of the problem can only be generated by an exhaustive search. Hence, for this problem solutions obtained from an algorithm will only be compared with solutions generated by a different algorithm.

4 Performance Evaluation Criteria

Zitzler et al. [14] suggest that in order to assess the optimality of non-dominated solutions identified by a multi-objective optimisation algorithm, these solutions should be compared with either the solutions obtained from a different algorithm or the true Pareto optimal solutions. Four corresponding measurement criteria are considered: the solution set coverage (C), the average distance between the non-dominated solutions to the Pareto optimal solutions (M_1), the distribution of the non-dominated solutions (M_2) and the normalised absolute difference between the extent of the front described by the non-dominated solutions and that obtained from the Pareto optimal solutions (M_3'). The solution set coverage is evaluated in the decision variable space while the remaining three criteria are calculated from the objective vectors of the solutions obtained. It is noted that the first three indices are taken directly from Zitzler et al. [14] while the final index is adapted from the M_3 index discussed in Zitzler et al. [14]. The M_3' index is introduced in this paper since the extent of the true Pareto front for each problem has a specific value and hence only the difference between the M_3 index calculated from the non-dominated solution set and that obtained from the true Pareto optimal solutions that yields a meaningful measurement [9].

5 Optimisation Results and Discussions

In this section, the results from using the evolutionary multi-objective optimisation algorithms (EMOAs) with the elitist strategy described in section 2.2 and the co-operative co-evolutionary multi-objective optimisation algorithms (CCMOAs) to solve T_1–T_6 problems and the topology design problem will be presented. The parameter setting for the algorithms that is used in all problems is displayed in Table 1. It is noted that after all repeated runs have completed, the final non-dominated solutions are then retrieved from either the individuals in the last generation from each run of a EMOA or the solutions in the preserved non-dominated solution set from every CCMOA run. The performance indicators introduced in section 4, i.e. M_1, M_2 and M_3', of each run are calculated and then averaged to obtain the final indices.

The M_1, M_2 and M_3' indices of the EMOAs and CCMOAs on the test problems T_1–T_6 are summarised in Table 2. In addition, the non-dominated solutions of the topology design problem in the objective space and the non-dominated solutions and the true Pareto optimal solutions of the test problems T_1–T_6 are displayed in Figs. 1–4 while the box plots of the C indices [14] from all seven problems are illustrated in Fig. 5. It is noted that the results displayed in Figs. 1 (right), 2, 3 and 4 (right) cover only the solutions from the MOGA and CCMOGA searches while Fig. 4 (left) illustrates only the solutions produced by the MOGA, CCMOGA and CCCNSGA. This is because most of the results from two different algorithms in the same category are very close to one another. Hence, the display of all search results would render the graphical representation useless and the displayed results are only used to present the overall effect of co-evolution.

From Table 2, in terms of the average distance from the identified non-dominated solutions to the true Pareto front as described by the M_1 criterion, a major improvement in the search performance of all algorithms can be observed in the test problems T_1, T_2, T_3, T_4 and T_6. In the test problem T_5, although the introduction of the co-operative co-evolutionary effect seems to produce only a minor improvement, this improvement is enough to push the solutions from the best deceptive Pareto front to the true Pareto front in the case of CCMOGA. This observation is confirmed by Fig. 4 (left). Nonetheless, even with the use of

Table 1. Parameter setting for the algorithms that is used in all problems. The fitness sharing, selection and elitist strategy listed here is not used in the implementation of CNSGA and CCCNSGA.

Parameter	EMOA	CCMOA
Chromosome representation	Binary chromosome	
Chromosome length of a complete solution	900 (T_1, T_2, T_3); 300 (T_4, T_6); 80 (T_5); 100 (Topology design problem)	
Fitness sharing	Triangular sharing function with the sharing radius estimated in the objective space [2]	
Selection method	Stochastic universal sampling or tournament selection (NPGA and CCNPGA only)	
Crossover method	Uniform crossover with probability = 1.0	
Mutation method	Bit-flip mutation with probability = 0.025	
Population/Sub-population size	200	200
Maximum size of preserved non-dominated solution set	-	50
Maximum number of elitist individuals	50	50
Number of generations	600	Number required for an equivalent number of objective evaluations
Number of repeated runs	30	30

Table 2. Summary of the EMOA and CCMOA performances on the test problems T_1–T_6. The neighbourhood parameters (σ) for the calculation of M_2 indices from normalised objective vectors are set to 0.04082. These parameters are set with the extent of the true Pareto front in the objective space as the guideline.

T	Index	MOGA	CCMOGA	NPGA	CCNPGA	NSGA	CCNSGA	CNSGA	CCCNSGA
T_1	M_1	0.1469	0.0000	0.1047	0.0002	0.1267	0.0001	0.0426	0.0002
	M_2	48.061	47.980	48.245	47.980	48.245	48.000	48.041	48.000
	M_3'	0.1629	0.0117	0.1542	0.0032	0.1937	0.0042	0.0805	0.0044
T_2	M_1	0.2033	0.0001	0.3237	0.0002	0.3156	0.0002	0.0608	0.0003
	M_2	43.244	48.000	25.255	47.959	27.571	47.959	47.980	47.960
	M_3'	0.0682	0.0015	0.0503	0.0006	0.0776	0.0006	0.0181	0.0003
T_3	M_1	0.0721	0.0000	0.0638	0.0001	0.0806	0.0001	0.0239	0.0001
	M_2	46.980	47.776	47.592	47.878	47.510	47.878	47.592	47.755
	M_3'	0.1194	0.0038	0.1093	0.0033	0.1084	0.0033	0.0468	0.0017
T_4	M_1	2.9706	0.0001	5.7623	0.0000	4.1912	0.0000	2.1231	0.0000
	M_2	47.082	48.041	11.309	48.000	27.537	48.041	48.674	48.041
	M_3'	1.8915	0.0047	2.5086	0.0016	4.5755	0.0021	0.7483	0.0012
T_5	M_1	0.0481	0.0456	0.0709	0.0712	0.0788	0.0661	0.0864	0.0847
	M_2	41.041	46.776	39.347	46.898	40.347	46.857	46.837	46.939
	M_3'	0.5754	0.2000	0.5876	0.3000	0.5267	0.3000	0.5000	0.4500
T_6	M_1	1.1317	0.0000	2.6990	0.0000	2.4099	0.0000	0.4288	0.0000
	M_2	17.134	48.000	5.0000	48.000	6.1670	48.000	37.010	47.959
	M_3'	0.1112	0.0000	0.3359	0.0000	0.2240	0.0000	0.0866	0.0000

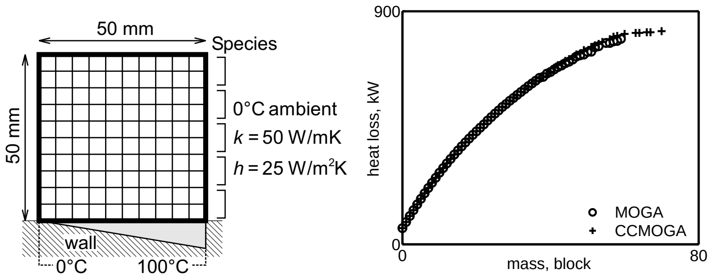

Fig. 1. Topology design problem (left) and the corresponding optimisation results (right).

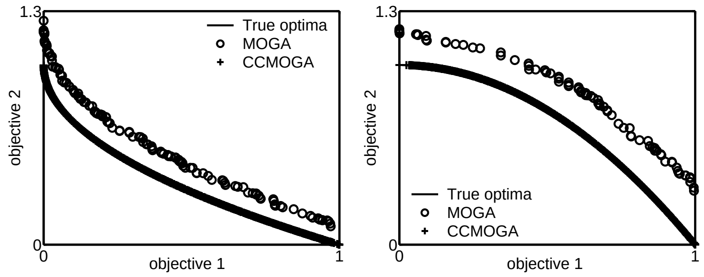

Fig. 2. Optimisation results of the T_1 (left) and T_2 (right) problems.

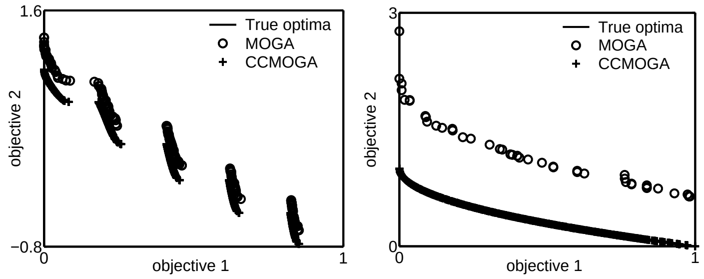

Fig. 3. Optimisation results of the T_3 (left) and T_4 (right) problems.

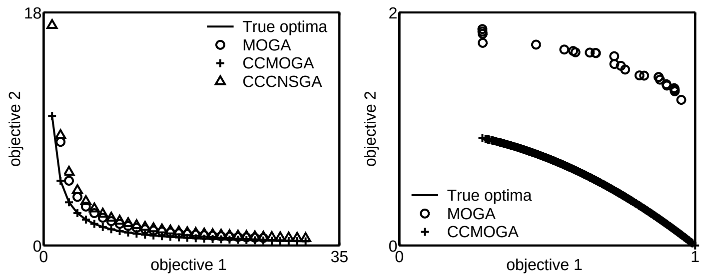

Fig. 4. Optimisation results of the T_5 (left) and T_6 (right) problems.

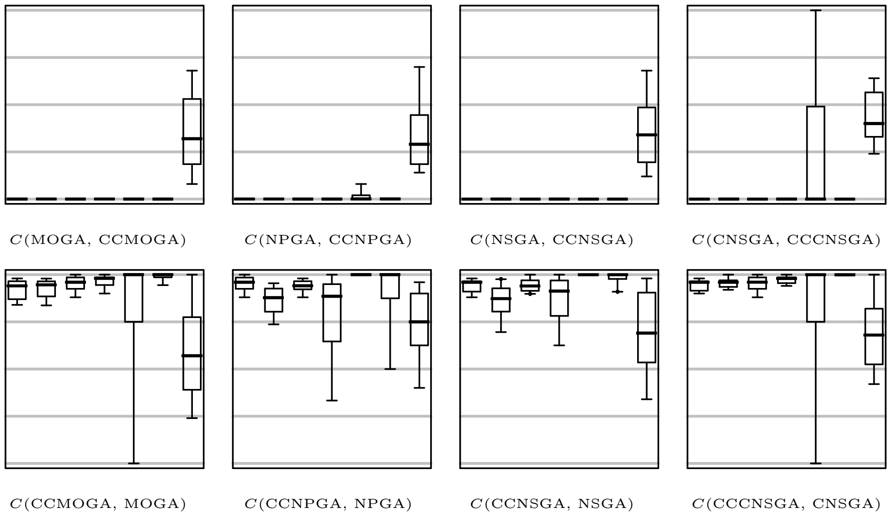

Fig. 5. Box plots of function $C(X, Y$ which is covered by $X)$ indices for each pair of EMOA and CCMOA. Each rectangle contains seven box plots representing the distribution of the C values; the leftmost box plot relates to T_1 while the rightmost to the topology design problem. The scale is 0 (no coverage) at the bottom and 1 (total coverage) at the top per rectangle.

the co-operative co-evolution, the CCNPGA, CCNSGA and CCCNSGA are still unable to locate the true Pareto optimal solutions of the test problem T_5.

Moving onto the consideration of the distribution of the identified non-dominated solutions. The M_2 indices in Table 2 indicate that the use of the co-operative co-evolution does not improve the solution distribution in the test problems T_1, and T_3. In contrast, a major improvement in the search results produced the CCNPGA and CCNSGA can be noticed in the test problems T_2, T_4, T_5 and T_6. In addition, some improvements can also be observed from the CCMOGA and CCCNSGA searches in the case of test problem T_6. In overall, the co-operative co-evolutionary effect helps creating a uniform distribution of solutions across the front.

In terms of the normalised absolute difference between the extent of the front described by the non-dominated solutions and that obtained from the true Pareto optimal solutions via the M_3' index, the co-operative co-evolutionary effect improves the performance of all algorithms. However, the search improvement in test problem T_5 is rendered less obvious numerically since the extent of the best deceptive Pareto front is very close to that of the true Pareto front. In other words, the M_3' index calculated from the solutions at the best deceptive Pareto front would only be slightly higher than that obtained from the solutions at the true Pareto front. For the test problem T_5, an additional use of graphical representation of solutions as shown in Fig. 4 (left) is required for the correct interpretation of the M_3' indices.

Finally, the solution set coverage using the C index is analysed. In contrast to the M_1, M_2 and M_3' indices, the C index can only be used to indicate whether the solution set obtained from one algorithm is dominated or equal to the solution set generated by another algorithm or not. The box plots illustrated in Fig. 5 clearly show that the solution sets obtained from the CCMOAs cover the solution sets generated from the corresponding EMOAs in the cases of T_1–T_6 problems. On the other hand, the solution sets obtained from the CCMOAs in the topology design problem only partially cover the solution sets generated from the counterpart EMOAs. This implies that the real-world problem considered is actually easier than the artificially generated benchmark problems. These results lead to the conclusion that although the EMOAs are not as efficient as the proposed CCMOAs, the EMOAs may still be good enough for the engineering problem considered.

6 Conclusions

In this paper, the effect of co-operative co-evolution on evolutionary multi-objective optimisation algorithms (EMOAs) has been investigated. The interested co-operative co-evolutionary effect is based on that described in a co-operative co-evolutionary genetic algorithm or CCGA [12, 13]. The co-operative co-evolutionary effect has been embedded into four genetic algorithms: a multi-objective genetic algorithm or MOGA [2, 5], a niched Pareto genetic algorithm or NPGA [3], a non-dominated sorting genetic algorithm or NSGA [4] and a controlled elitist non-dominated sorting genetic algorithm or CNSGA [7]. Subsequently, the co-operative co-evolutionary multi-objective optimisation algorithms (CCMOAs) have been benchmarked against the EMOAs in seven test problems. The first six problems cover different characteristics of multi-objective optimisation problems, namely convex Pareto front, non-convex Pareto front, discrete Pareto front, multi-modality, deceptive Pareto front and non-uniformity of solution distribution [14]. In contrast, the last problem is a two-objective real-world problem, which is generally referred to as a topology design [15]. The simulation results indicate that in general the CCMOAs are superior to the EMOAs. This conclusion is based upon the solution set coverage, the average distance from the non-dominated solutions and the true Pareto front, the distribution of the non-dominated solutions and the extent of the front described by the non-dominated solutions [14].

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